Indian J. Phys. **71B** (2), 133–146 (1997)

IJP B
— an international journal

Positronium formation in the laser assisted collision of positron with hydrogen atom in the first excited state

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Received 11 September 1996, accepted 18 December 1996

Abstract : A straightforward method for the study of positronium formation in the ground state in the laser assisted positron-hydrogen atom collisions, the target hydrogen atom being in the first excited state ($n = 2$), is presented in the framework of the first Born approximation (FBA). The laser field is treated classically as a homogeneous, single mode and linearly polarised electric field while the collision dynamics is treated quantum mechanically. The projectile-field interaction and the dressing effects on the hydrogen and positronium atoms have been considered to first order. The dressed wave functions have been constructed adopting the $\boldsymbol{A}.\boldsymbol{p}$ gauge. The FBA rearrangement scattering amplitudes considering the full interaction have been obtained in closed analytic forms for field direction (ε_0) parallel to the incident positron momentum ($\boldsymbol{k}_i$) *i.e.* $\varepsilon_0 \parallel \boldsymbol{k}_i$. The differential cross sections for Ps formation have been presented for no photon and one photon exchange ($l = 0, \pm 1$) at 50, 100 and 200 eV incident energies for laser field parameters $\varepsilon_0 = 0.00002$ a.u. and $\omega = 0.0043$ a.u. along with the corresponding field free results. The variations of the differential cross sections with laser field strength, frequency and incident positron energy have also been shown.

Keywords : Positronium, laser field, hydrogen atom in excited state

PACS Nos. : 34.80.Qb, 34.70.+e, 32.60.+i

1. Introduction

With the tremendous advancements in laser technology, laser-assisted collision processes are becoming more and more attractive for both the theoretical and the experimental workers. Systematic studies on laser-assisted electron-atom collisions have been made in a series of papers, by Joachain and his coworkers [1–3]. Recently, Born-Floquet theory has been proposed [4] to study such collision processes. In a very recent work, the polarisation dependence of laser assisted electron-atom elastic collisions have been investigated by Fainstein and Maquet [5].

A detailed quantum-mechanical analysis has been given by Caggegi *et al* [6] for the rearrangement collisions of structureless projectiles and hydrogenic targets in the presence of a laser field, treated classically in the dipole approximation. However as pointed out by them, accurate numerical calculations using their derived formulae are extremely difficult and as such simpler alternative versions are needed.

Bhattacharya *et al* [7] have dealt with such a simplified treatment of the specific rearrangement collision process of positronium (Ps) formation in the ground state, in positron-hydrogen-atom collision in the presence of a laser field, using the $\boldsymbol{E}.\boldsymbol{r}$ gauge. Further simplification in the reduction of the rearrangement scattering amplitude for hydrogen and helium as target atoms has been carried out recently by Li Shu-Min *et al* [8], adopting the $\boldsymbol{A}.\boldsymbol{p}$ gauge.

In the present study, we have investigated in the $\boldsymbol{A}.\boldsymbol{p}$ gauge, Ps formation in the ground state in the laser-assisted positron-hydrogen atom collisions, where the target hydrogen atom is in the first excited state ($n = 2$). As far our knowledge goes, no theoretical calculation has yet been done for Ps formation by capturing electron from an excited state of the target atom in the presence of a laser field. As a first attempt, the calculations have been performed in the framework of the first Born approximation (FBA) considering the direction of the electric field ε_0 as the polar axis. It has been noted that the first Born approximation yields a reasonable result for Ps formation in the field-free e^+–H collision in the intermediate and high energy region, which is our main concern for the present study. Even for energies above 20 eV, it seems from the work of Weber *et al* [9] that the first Born calculation for Ps formation in field-free e^+–H collisions gives results as good as those obtained from more sophisticated theories. A clear account of the field-free positron atom collisions is given in a recent review by Ward [10]. The relative simplicity of the analytical treatment by the first Born approximation is very suitable for a qualitative understanding of the main features of the scattering processes in which we are interested.

The $\boldsymbol{A}.\boldsymbol{p}$ gauge is adopted in the present investigation because here it has a distinct advantage over the $\boldsymbol{E}.\boldsymbol{r}$ gauge in treating the rearrangement collisions namely the $\boldsymbol{A}.\boldsymbol{p}$ gauge simplifies greatly the reduction of the S-matrix and the scattering amplitudes are obtained in closed analytic forms. It should be noted however, that though for the exact wave functions the gauges are equivalent, this is not the case with approximate wave functions.

The laser field has been treated classically as a homogeneous, single mode and linearly polarised electric field and the frequency of the field is considered to be low with respect to typical atomic excitation energies. The laser modified wave functions of the projectile and the outgoing free Ps atom are given by the Volkov solutions. The dressed bound state wave functions are obtained by solving the Schrödinger equation in the $\boldsymbol{A}.\boldsymbol{p}$ gauge through first order time-dependent perturbation theory, assuming the field strength ε_0 to be much less than the characteristic intra-atomic electric field. It may be pointed out here that in applying first order time-dependent perturbation theory, difficulty arises due to l-degeneracy of the $n = 2$ level of the H atom. In order to avoid this difficulty, we have used combinations of $2S$ and $2P_0$ states as the unperturbed wave functions. The introduction of the laser field splits the unperturbed $n = 2$ level of hydrogen atom into three sub-levels; one is shifted above and the other below with respect to the unperturbed level by an equal amount and they correspond to $(2S - 2P_0)$ and $(2S + 2P_0)$ states, respectively, while the remaining ones corresponding to $2P_{\pm 1}$ are unaltered.

Using these dressed wave functions, we have arrived at closed form expressions for the positronium formation cross sections.

2. Theory

Let us consider the following laser assisted collision process :

$$e^+ + \mathrm{H}\,(n=2) \rightarrow \mathrm{Ps}\,(1S) + \mathrm{H}^+. \tag{1}$$

We shall assume that the laser field is treated classically as a spatially homogeneous, single mode and linearly polarised electric field,

$$\varepsilon(t) = \varepsilon_0 \sin(\omega t), \tag{2}$$

the corresponding vector potential in the $\boldsymbol{A}.\boldsymbol{p}$ gauge being

$$A(t) = A_0 \cos(\omega t) \tag{3}$$

with $\qquad A_0 = c\varepsilon_0 / \omega.$

The first Born S-matrix element (in atomic units) for Ps formation in the ground state is given by

$$S_{fi}^{\mathrm{Ps}} = -i \int_{-\infty}^{+\infty} dt \left\langle \Psi_f | V | \Psi_i \right\rangle, \tag{4}$$

where $\qquad V = 1/r_2 - 1/r_1 \qquad (5)$

is the post form of the field free interaction and

$$\Psi_i = \chi_{k_i}(\boldsymbol{r}_2, t)\Phi_d^{\mathrm{H}(n=2)}(\boldsymbol{r}_1, t), \tag{6a}$$

and

$$\Psi_f = \chi_{k_f}(\boldsymbol{s}, t)\Phi_d^{\mathrm{Ps}}(\boldsymbol{r}_{12}, t) \tag{6b}$$

are the initial and final state wave functions, respectively.

r_1 and $\boldsymbol{r}_2$, in the above equations, are the position vectors of the electron and the positron respectively while $\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and $\boldsymbol{s} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$. In eqs. (6), $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ are respectively, the wave vectors of the incident positron and the outgoing free Ps atom in the final state. χ_{k_i} and χ_{k_f} in eqs. (6) are the Volkov solutions representing the non-relativistic laser modified wave functions (normalised to a delta function) for the projectile and the outgoing free Ps atom and are given by

$$\chi_{k_i}(\boldsymbol{r}_2, t) = (2\pi)^{-3/2} \exp[i\{\boldsymbol{k}_i . \boldsymbol{r}_2 + \boldsymbol{k}_i . \boldsymbol{\alpha}_0 \sin \omega t - k_i^2 t / 2\}] \tag{7}$$

with $\qquad \alpha_0 = \varepsilon_0 / \omega^2$

and

$$\chi_{k_f}(\boldsymbol{s}, t) = (2\pi)^{-3/2} \exp[i\{\boldsymbol{k}_f . \boldsymbol{s} - k_f^2 t / 4\}]. \tag{8}$$

It may be noted that in eq. (8), the effect of the laser field on the motion of the Ps atom with respect to the target nucleus, cancels out because of the equal and opposite charges of the constituents of the Ps atom. In writing the Volkov solutions, we have omitted the terms involving A^2 since we restrict ourselves to first order in the laser field strength.

The dressed excited state wave functions of the target hydrogen atom $\Phi_d^{H(n=2)}$ in eq. (6a), is obtained by solving the coupled Schrödinger equation in the $\boldsymbol{A.p}$ gauge using the time-dependent first order perturbation theory following the work of Li Shu-Min *et al* [8]. In the linear Stark effect as a result of the static external field, the unperturbed $n = 2$ level of the hydrogen atom splits into three levels : one is shifted above and the other below by an equal amount with respect to the unperturbed level and correspond to $(2S - 2P_0) \equiv \Phi_0^{H(100)}$ and $(2S + 2P_0) \equiv \Phi_0^{H(010)}$ states respectively, while the remaining ones corresponding to $(2P_{\pm 1}) \equiv \Phi_0^{H(00\pm 1)}$ states are unaltered. For the present investigation, we assume the laser frequency to be very small so that the laser field may be considered to be quasi-static. Thus we may write

$$\Phi_d^{H(100)}(\boldsymbol{r}_1,t) = \exp\left[-iW_1^{H}t\right]\left[\phi_0^{H(100)}(\boldsymbol{r}_1) - \cos\omega t\,\tilde{\phi}_0^{H(100)}(\boldsymbol{r}_1)\right] \tag{9}$$

where

$$\phi_0^{H(100)}(\boldsymbol{r}_1) = \frac{1}{4}(\pi)^{-1/2}e^{-\lambda_1 r_1}\left[1 - r_1(1+\cos\theta_1)/2\right], \tag{10}$$

and

$$\tilde{\phi}_0^{H(100)}(\boldsymbol{r}_1) = \frac{1}{4}(\pi)^{-1/2}e^{-\lambda_1 r_1}\left[i\lambda_1\varepsilon_0\cdot\hat{r}_1\left\{1 - r_1(1+\cos\theta_1)/2\right\}\right.$$
$$\left. + i\varepsilon_0\cdot\hat{r}_1/2 + i\varepsilon_0/2\right]\left[\overline{\omega}_{H(100)}\omega\right]. \tag{11}$$

In eq. (9), $W_1^{H} = -1/8$ a.u. is the energy of the unperturbed first excited state of the H atom. In eqs. (10) and (11), $\lambda_1 = 1/2$, $\overline{\omega}_{H(100)} = 4/21$ a.u. is the average excitation energy of the dressed hydrogen atom in the excited state and $\hat{r}_1$ is the unit vector of $\boldsymbol{r}_1$.

The necessary criterion for the perturbation theory to be valid, may be written as

$$\frac{\left|\left\langle\phi_{n'}^{H}(\boldsymbol{r}_1)\right\rangle\left|\frac{A_0}{c}\cdot\boldsymbol{p}\right|\left\langle\phi_{n(=2)}^{H}(\boldsymbol{r}_1)\right\rangle\right|}{\left|\omega_{nn'} - \omega\right|} << 1 \tag{12}$$

for a pair of states $\phi_{n(=2)}^{H}(r_1)$ and $\phi_{n'}^{H}(r_1)$ with transition frequency $\omega_{nn'}$.

The range of validity of the theory on the laser field strength and frequency can be obtained from eq. (12). In our case, for the particular frequency used, for example $\hbar\omega = 0.117$ eV, the field strength ε_0 should not exceed 1.0×10^5 V cm^{-1}.

It may be pointed out that for degenerate states, the energy difference term ω_{nn} in the denominator of eq. (12) happens to be almost zero except for the laser frequency term which is a very small quantity. Hence for the degenerate states, the criterion for validity of the perturbation theory will not be satisfied. In order to avoid this difficulty, we have adopted the usual procedure. Combinations of $2S$ and $2P_0$ wave functions have been taken as the unperturbed wave functions. As a result, the matrix element in the numerator of eq. (12) is identically zero for all the degenerate final states of $n = 2$ level so that the validity criterion [eq. (12)] is satisfied. It should be noted that the small energy differences due to the splitting of the $n = 2$ level of hydrogen atom on account of the weak laser field is not of much significance in the entire calculation done in the spirit of the first order perturbation theory.

In a similar fashion, Φ_d^{Ps}, the dressed ground state wave function of the Ps atom formed in the final channel [eq. (6b)] has been constructed :

$$\Phi_d^{\mathrm{Ps}}(r_{12},t) = \exp\left[-iW_0^{\mathrm{Ps}}t\right]\left[\phi_0^{\mathrm{Ps}}(r_{12}) - \cos\omega t\,\tilde{\phi}_0^{\mathrm{Ps}}(r_{12})\right], \tag{13}$$

where
$$\phi_0^{\mathrm{Ps}}(r_{12}) = \frac{1}{2\sqrt{2}}(\pi)^{-1/2}e^{-\lambda_2 r_{12}} \tag{14}$$

and
$$\tilde{\phi}_0^{\mathrm{Ps}}(r_{12}) = i\lambda_2\varepsilon_0.\hat{r}_{12}\,\phi_0^{\mathrm{Ps}}(r_{12})/\left[\overline{\omega}_{Ps}\omega\right]. \tag{15}$$

In the above equations, $W_0^{\mathrm{Ps}} = -\frac{1}{4}$ a.u., $\lambda_2 = -1/2$, $\overline{\omega}_{\mathrm{Ps}} = 2/9$ a.u. is the average excitation energy of the dressed Ps atom and $\hat{r}_{12}$ is the unit vector of r_{12}.

We shall now give a brief outline of our approach for the specific transition $\Phi_d^{\mathrm{H(100)}} \rightarrow \Phi_d^{\mathrm{Ps}}$.

In view of eqs. (4), (7), (8), (9) and (13) the transition matrix element is given by

$$S_{fi}^{\mathrm{Ps}} = -i\int_{-\infty}^{+\infty} dt\left\langle \chi_{k_f}(s,t)\Phi_d^{\mathrm{Ps}}(r_{12},t)\left|\left(\frac{1}{r_2}-\frac{1}{r_1}\right)\right|\chi_{k_i}(r_2,t)\Phi_d^{\mathrm{H(100)}}(r_1,t)\right\rangle. \tag{16}$$

Following Bhattacharya *et al* [7], the time integration in the above equation is readily carried out with the help of addition theorem of Bessel functions. It may be mentioned here that an additional term exp $[-i\cos(\epsilon t)]$ has been inserted in the integrand in order to generate, by parametric differentiation with respect to ϵ, the terms involving cos (ωt) in the dressing parts of the bound state wave functions of the hydrogen and the positronium atoms [see eqs. (9) and (13)]. Final results of course, will correspond to the limiting value as $\epsilon \rightarrow 0_+$. Thus after time integration, the transition matrix element may be written as

$$S_{fi}^{\mathrm{Ps}} = -i(2\pi)^{-2}\sum_{-\infty}^{+\infty}\delta(W_0^{\mathrm{Ps}} + k_f^2/4 - W_1^{\mathrm{H}} - k_i^2/2 + \omega l)F_l, \tag{17}$$

where l is the number of photon exchanged. $l > 0$ means emission of photons while $l < 0$ corresponds to absorption. The energy conservation relation is given by the delta function

$$W_1^{\mathrm{H}} + k_1^2/2 = W_0^{\mathrm{Ps}} + k_f^2/4 + \omega l;\ l = 0,\pm 1,\pm 2,\ldots \tag{18}$$

In eq. (17), the term F_l is the transition amplitude and is given by

$$F_l = J_l(k_i.\alpha_0)\Big[f_l(\phi_0^{H(100)} \to \phi_0^{Ps}) - l(k_i.\alpha_0)^{-1}\Big\{f_l\Big(\tilde{\phi}_0^{H(100)} \to \phi_0^{Ps}\Big)$$

$$+ f_l\Big(\phi_0^{H(100)} \to \tilde{\phi}_0^{Ps}\Big)\Big\}\Big]\Big/\big[8\sqrt{2}\pi\big], \tag{19}$$

where $f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right)$, $f_l\left(\tilde{\phi}_0^{H(100)} \to \phi_0^{Ps}\right)$, and $f_l\left(\phi_0^{H(100)} \to \tilde{\phi}_0^{Ps}\right)$ are respectively, the FBA amplitudes corresponding to the tansitions $\phi_0^{H(100)} \to \phi_0^{Ps}$, $\tilde{\phi}_0^{H(100)} \to \phi_0^{Ps}$ and $\phi_0^{H(100)} \to \tilde{\phi}_0^{Ps}$ and $J_l\left(k_i.\alpha_0\right)$ is the Bessel function of order l.

Now, each of this amplitudes in eq. (19) has to be evaluated. First, consider $f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right)$. With the help of eqs. (10) and (14) we have

$$f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right) = -(\pi)^{-1}\iint d^3r_1 d^3r_2\, e^{-i\chi_1\cdot r_1}\, e^{-i\chi_2\cdot r_2}$$

$$e^{-\lambda_1 r_1}\, e^{-\lambda_2 r_{12}}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\left[1 - r_1(1+\cos\theta_1)/2\right], \tag{20}$$

where $\chi_1 = k_f/2$ and $\chi_2 = k_f/2 - k_i$. Similar type of expressions will be obtained for the remaining amplitudes.

The space integrations in the above equations can be performed easily (see Appendix) and finally the amplitude $f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right)$in eq. (20) may be obtained as

$$f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right) = -(\pi)^{-1}\left[\frac{\partial}{\partial\lambda_2}(I_0) + \frac{1}{2}\frac{\partial^2}{\partial\lambda_1\partial\lambda_2}(I_0) - \frac{1}{2}i\frac{\partial^2}{\partial\chi_{1z}\partial\lambda_2}(I_0)\right.$$

$$\left. + \frac{\partial^2}{\partial\lambda_1\partial\lambda_2}(I_2) + \frac{1}{2}\frac{\partial^3}{\partial\lambda_1^2\partial\lambda_2}(I_2) - \frac{1}{2}i\frac{\partial^3}{\partial\chi_{1z}\partial\lambda_1\partial\lambda_2}(I_2)\right]. \tag{21}$$

In a similar fashion, other two amplitudes $f_l\left(\tilde{\phi}_0^{H(100)} \to \phi_0^{Ps}\right)$ and $f_l\left(\phi_0^{H(100)} \to \tilde{\phi}_0^{Ps}\right)$ in eq. (19) can be obtained. Therefore in view of eq. (19), the final transition amplitude F_l is reduced to an exact analytic form.

In view of eq. (19), the differential cross section with the transfer of l photons between the laser field and the scattering system may be written as

$$\left.\frac{d\sigma_l}{d\Omega}\right)_{\Phi_d^{H(100)} \to \Phi_d^{Ps}} \equiv \left(\frac{d\sigma_l}{d\Omega}\right)_{100} = \frac{k_f}{2k_i}\left|F_l\right|^2. \tag{22}$$

Transition amplitudes for transitions to the other excited states of the $n = 2$ level of the hydrogen atom, $\Phi_d^{H(010)}$ and $\Phi_d^{H(00\pm1)}$, may also be deduced following the same procedure as discussed in the previous case. Now for symmetry reasons, contributions to the differential cross sections due to the $\Phi_d^{H(100)}$ and $\Phi_d^{H(010)}$ states are found to be identical. Therefore, the differential cross sections for Ps formation in the ground state in laser-assisted e$^+$–H ($n = 2$) collision may be written as

$$\left(\frac{d\sigma_l}{d\Omega}\right)_{\Phi_d^{H(n=2)} \to \Phi_d^{Ps}} = 2\left[\left(\frac{d\sigma_l}{d\Omega}\right)_{100} + \left(\frac{d\sigma_l}{d\Omega}\right)_{001}\right] \tag{23}$$

3. Results and discussion

The positronium formation differential cross section for laser-assisted e^++H ($n = 2$) collisions have been studied taking account of the dressing effects of H and Ps atoms for field direction parallel to the incident positron momentum *i.e.* $\varepsilon_0 \parallel \boldsymbol{k}_i$.

The differential cross sections for Ps formation in the ground state have been displayed in Figure 1 as functions of the scattering angle for laser field parameters $\varepsilon_0 = 1.0 \times 10^5$ V cm^{-1}, $\hbar\omega = 0.117$ eV and incident energy $E_i = 50$ eV with two different

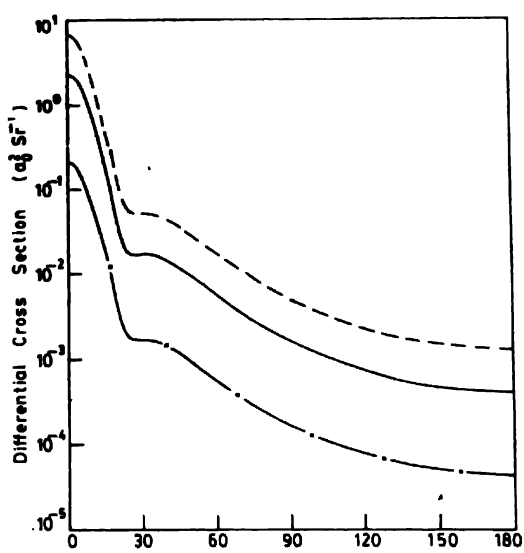

Figure 1. The first Born differential cross sections ($a_0^2 sr^{-1}$) for Ps formation in the ground state in the laser assisted e$^+$ + H ($n = 2$) collisions with $\varepsilon_0 \parallel \boldsymbol{k}_i$ for no photon exchange ($l = 0$) (—·—·—) and one photon absorption ($l = -1$) (———) at 50 eV incident energy with $\varepsilon_0 = 1.0 \times 10^5$ V cm^{-1} and $\hbar\omega = 0.117$ eV along with the corresponding field free cross sections (- - - -).

l values, $l = 0$ (no photon exchange) and $l = -1$ (one photon absorption) along with the corresponding field-free cross section results. The curve corresponding to $l = +1$ (one photon emission) can not be distinguished from those corresponding to $l = -1$ in scale and as such, we have not shown them in this figure. The qualitative behavior for each of the three curves in Figure 1 is more or less the same. Each of the curves falls sharply from a forward peak to a minimum, then rises again to a maximum followed by monotonic fall with increasing scattering angle. But quantitatively they differ remarkably. Figure 1 shows that the curves with the laser field switched on are much below the corresponding field-free curve which reflects that the introduction of the laser field reduces the cross section values quite significantly. The curve corresponding to $l = 0$ is always below the curve for $l = -1$ which demonstrates that Ps formation is much favoured with the absorption of photons than with no photon exchange.

Figures 2 and 3 represent the same as Figure 1 but corresponding to different incident energies $E_i = 100$ eV and $E_i = 200$ eV, respectively. The qualitative nature of the

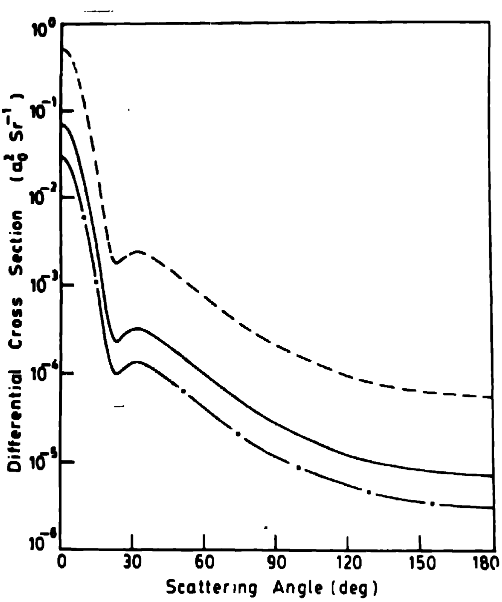

Figure 2. Same as of Figure 1 but for 100 eV incident energy.

curves in the Figures 2 and 3 is almost similar with the curves in Figure 1. However, all the curves become more peaked in the forward direction as the incident energy increases. Regarding quantitative behavior, a sharp difference is observed between the curves in Figures 2 and 3 and that in Figure 1. It is seen that the curve for $l = 0$ in

Figure 3 lies above the corresponding curve for $l = -1$ while the situation is reverse in Figures 1 and 2. Thus, it may be concluded that the possibility of Ps formation is considerably enhanced with $l = 0$ than with $l = -1$ which is just the opposite situation compared to Figures 1 and 2. It may be mentioned here that for no photon exchange ($l = 0$), the dressing effects of H and Ps atoms are zero. Comparing the curves in Figures 1, 2 and 3, it is noted that the positions of the minima of the curves shift towards smaller angles as the incident energy increases.

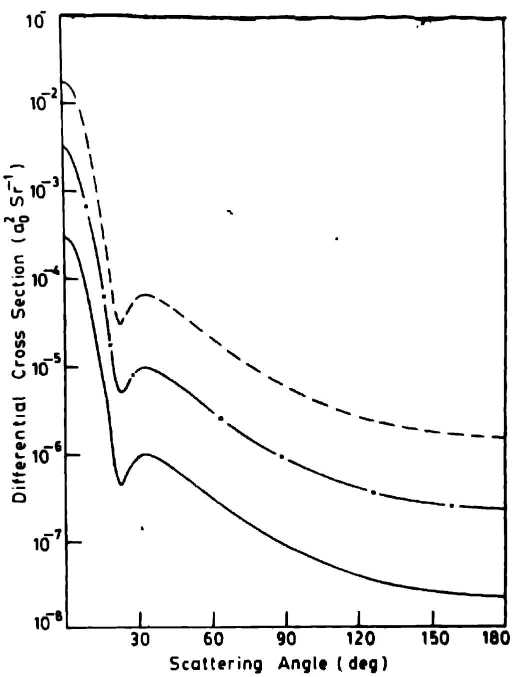

Figure 3. Same as of Figure 1 but for 200 eV incident energy.

The variations of the Ps formation differential cross sections with incident energy, laser field strength and laser frequency are shown in Figures 4, 5 and 6 respectively for $\Theta = 10°$ and $l = -1$. From Figure 4, it is seen that for the adopted values of the laser field parameters $\varepsilon_0 = 1.0 \times 10^5$ V cm^{-1} and $\hbar\omega = 0.117$ eV, the cross section falls monotonically with increasing incident energy until a dip minimum is attained near 170 eV and then rises with incident energy. In Figure 5, it is observed that the cross section value increases sharply starting from a minimum, reaches a maximum and then falls monotonically as the field strength increases from $\varepsilon_0 = 1.0 \times 10^3$ V cm^{-1} to $\varepsilon_0 = 1.0 \times 10^5$ V cm^{-1} with $\hbar\omega = 0.117$ eV, and $E_i = 100$ eV. From Figure 6, it is seen that the differential cross section increases with a number of maxima and minima as the laser frequency changes from $\hbar\omega = 0.037$ eV, to $\hbar\omega = 0.117$ eV with $\varepsilon_0 = 1.0 \times 10^5$ V cm^{-1} and $E_i = 100$ eV. Moreover, the lower the laser frequency, the more the number of maximum and zero and the separation between two consecutive maxima goes on decreasing with decreasing frequency. The explanation for this kind of nature of the curve may be as follows :

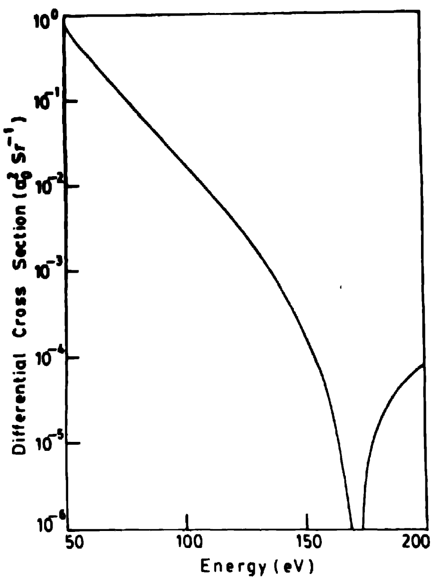

Figure 4. Differential cross sections (a_0^2sr^{-1}) as functions of incident energy (eV) for $l = -1$, $\theta = 10°$, $\varepsilon_0 = 1.0 \times 10^5$ V cm^{-1} and $\hbar\omega = 0.117$ eV.

The presence of the Bessel function in eq. (19) is responsible for the observed maxima and zeros in the curve. It is known from the nature of the Bessel functions that the relative separation of the zeros of the Bessel function decreases gradually with increasing value of the argument and finally the separation becomes almost constant. Now, it is found in the preceeding section that the argument of the Bessel function varies inversely with the square of the laser frequency. Therefore, the argument of the Bessel function decreases with increaing laser frequency. As a result, the number of maximum and zero decreases with increasing separation between two consecutive maxima as is displayed in Figure 6.

No theoretical calculation has yet been performed for Ps formation in the presence of a laser field by capturing an electron from the excited state of the target hydrogen atom by the incident positron. Thus, we are not in a position to compare our results with other theoretical findings. However, we have calculated separately the 2*S* and 2*P* field-free Ps formation cross section for such process and as a check, we have reproduced these cross sectión results.

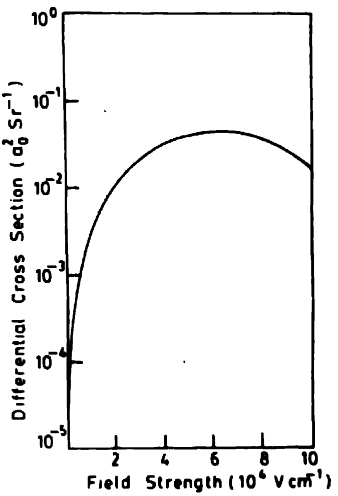

Figure 5. Differential cross sections ($a_0^2 sr^{-1}$) as functions of laser field strength (V cm^{-1}) for $l = -1$, $\theta = 10°$, $\hbar\omega = 0.117$ eV and $E_i = 100$ eV.

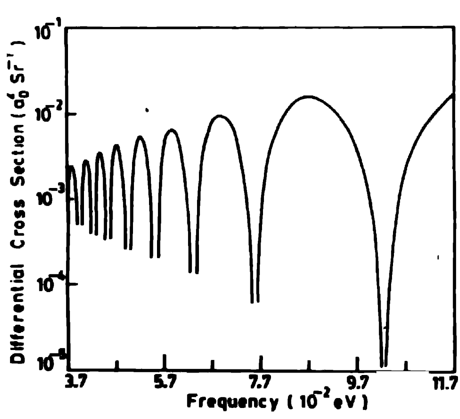

Figure 6. Differential cross sections ($a_0^2 sr^{-1}$) as functions of laser frequency (eV) for $l = -1$, $\theta = 10°$, $\varepsilon_0 = 1.0 \times 10^5$ V cm^{-1} and $E_i = 100$ eV.

4. Summary

In this work, we have developed in the framework of the first Born approximation, a suitable method for the evaluation of the Ps formation differential cross sections for a laser assisted rearrangement collision where the incident positron captures an electron from the target hydrogen atom which is in the first excited state ($n = 2$). It may be pointed out that due to l-degeneracy of the $n = 2$ level of the target H atom, difficulty arises in obtaining the dressed excited state wave functions of the H atom empolying first order time-dependent perturbation theory. In order to overcome this difficulty successfully, appropriate combinations of $2S$ and $2P_0$ states have been taken as unperturbed states. This is due to the fact that as the laser field is introduced the unperturbed $n = 2$ level splits into three sub-levels : one is shifted above and the other below with respect to the unperturbed $n = 2$ level by an equal amount and correspond to $(2S - 2P_0)$ and $(2S + 2P_0)$ states, respectively, while the remaining ones corresponding to $2P_{\pm 1}$ states are unaltered.

The salient feature of the given method in the $A.p$ gauge is that the expressions of the FBA amplitudes for transitions from different Stark states of the target H atom to the ground state of the Ps atom for a particular process (*e.g.* $l = 0$ or $l = \pm 1$), does not involve any integration to be carried out numerically as in the work of Li Shu-Min *et al* [8] for Ps formation in laser-assisted $e^+ + H$ $(1S)$ collisions. On the contrary, the transition amplitudes for a particular process have been obtained in closed analytic forms without resorting to any further approximation. Therefore, we expect that the present approach will be more advantageous for the evaluation of the rearrangement scattering amplitudes.

The results presented for the Ps formation in the ground state in laser-assisted e^++H ($n = 2$) collisions are quite new and as such no comparative study is possible.

Acknowledgment

One of the authors (MB) would like to thank the Council of Scientific and Industrial Research, India, for providing partial financial support.

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Appendix

In this mathematical appendix we describe the methods of evaluation of the FBA amplitudes in eq. (19).

For the evaluation of the amplitude $f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right)$ in eq. (20), we consider the following mother integrals

$$I_0 = \iint d^3r_1 d^3r_2\, e^{-i\chi_1 \cdot r_1}\, e^{-i\chi_2 \cdot r_2}\, \frac{e^{-\lambda_1 r_1}}{r_1}\, \frac{e^{-\lambda_2 r_{12}}}{r_{12}} \tag{A1}$$

and
$$I_2 = \iint d^3r_1 d^3r_2\, e^{-i\chi_1 \cdot r_1}\, e^{-i\chi_2 \cdot r_2}\, \frac{e^{-\lambda_1 r_1}}{r_1}\, \frac{e^{-\lambda_2 r_{12}}}{r_{12}}\, \frac{1}{r_2}. \tag{A2}$$

Using the Fourier integral representations for $\left[e^{-\lambda_1 r_1} / r_1\right]$ and $\left[e^{-\lambda_2 r_{12}} / r_{12}\right]$ and the properties of Dirac delta function, the integration of eq. (A1) may be readily carried out and we get

$$I_0 = \frac{16\pi^2}{\left[\chi_2^2 + \lambda_2^2\right]\left[\left|\chi_1 + \chi_2\right|^2 + \lambda_1^2\right]}. \tag{A3}$$

In a similar way, the integral I_2 in eq. (A2) may be reduced to the following form

$$I_2 = 8\int d^3q \left[\left(\left|q + \chi_2\right|^2\right)\left(\left|q - \chi_1\right|^2 + \lambda_1^2\right)\left(q^2 + \chi_2^2\right)\right]^{-1}, \tag{A4}$$

which is the wellknown Lewis' integral and the result is given by

$$I_2 = 8\pi^2\left(\beta^2 - \alpha\gamma\right)^{-1/2} \log\left[\frac{\beta + (\beta^2 - \alpha\gamma)^{1/2}}{\beta - (\beta^2 - \alpha\gamma)^{1/2}}\right], \tag{A5}$$

where
$$\beta = \lambda_2\left[\left|\chi_1 + \chi_2\right|^2 + \lambda_1^2\right] + \left[\chi_2^2 + \lambda_2^2\right]\lambda_1, \tag{A6}$$

$$\alpha\gamma = \left[\left|\chi_1 + \chi_2\right|^2 + \lambda_1^2\right]\left[\chi_2^2 + (\lambda_1 + \lambda_2)^2\right]\left[\chi_2^2 + \lambda_2^2\right]. \tag{A7}$$

Thus, the amplitude $f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right)$ in eq. (20) can now be obtained by suitable parametric differentiations of I_0 and I_2 :

$$f_l\left(\phi_0^{H(100)} \to \phi_0^{Ps}\right) = -(\pi)^{-1}\left[\frac{\partial}{\partial\lambda_2}(I_0) + \frac{1}{2}\frac{\partial^2}{\partial\lambda_1\partial\lambda_2}(I_0) - \frac{1}{2}i\frac{\partial^2}{\partial\chi_{1z}\partial\lambda_2}(I_0)\right.$$
$$\left. + \frac{\partial^2}{\partial\lambda_1\partial\lambda_2}(I_2) + \frac{1}{2}\frac{\partial^3}{\partial\lambda_1^2\partial\lambda_2}(I_2) - \frac{1}{2}i\frac{\partial^3}{\partial\chi_{1z}\partial\lambda_1\partial\lambda_2}(I_2)\right]. \tag{A8}$$

In a similar fashion, other two amplitudes $f_l\left(\tilde{\phi}_0^{\mathrm{H(100)}} \rightarrow \phi_0^{\mathrm{Ps}}\right)$ and $f_l\left(\phi_0^{\mathrm{H(100)}} \rightarrow \tilde{\phi}_0^{\mathrm{Ps}}\right)$ in eq. (19) can be obtained. But in the evaluation $f_l\left(\tilde{\phi}_0^{\mathrm{H(100)}} \rightarrow \phi_0^{\mathrm{Ps}}\right)$, another mother integral I_1 has to be considered.

$$I_2 = \iint d^3r_1 d^3r_2\, e^{-i\chi_1 \cdot r_1}\, e^{-i\chi_2 \cdot r_2}\, \frac{e^{-\lambda_1 r_1}}{r_1}\, \frac{e^{-\lambda_2 r_{12}}}{r_{12}}\, \frac{1}{r_1}. \tag{A9}$$

Following similar procedure as is done in the evaluation of I_2 [eq. (A2)], this integral is evaluated to give

$$I_1 = \frac{16\pi^2}{|\chi_1 + \chi_2|\left[\chi_2^2 + \lambda_2^2\right]} \tan^{-1}\left\{\frac{|\chi_1 + \chi_2|}{\lambda_1}\right\}. \tag{A10}$$